Wei Xian Lim 901579699

1. (a) Proof by contrapositive: F ∘ G and G one-to-one don't ensure F is.

As a counter-example, let A = {1}, B = {1, 2}, C = {1}, and g : A 🡪 B where g(1) = 1 and f : B 🡪 C where f(1) = f(2) = 1. Then F ∘ G  : A 🡪 C is defined by (F ∘ G )(1) = 1. This map is a bijection from A = f1g to C = f1g, so is one-to-one. However, F is not

one-to-one, since F(1) = F(2) = 1.

(b) Suppose x, y ∈ A and g(x) = g(y). Therefore F ∘ G (x) = F ∘ G (y) 🡪 x = y because F ∘ G is one-to-one

1. (a) it is anti-reflexive. We cannot have (1, -1) because (1, -1) ≱ 0

it is symmetric. xy = yx ∈ R1

it is transitive. We have (x, y) ∈ R1 and (y, z) ∈ R1

if xy ≥ 0, both x and y ≥ 0. So all x, y, z are ≥ 0.

Therefore, it follows that (x, z) ∈ R1

(b) It is anti-reflexive. We cannot have (2, 2) because it doesn’t satisfy the statement

x = 2y.

It is anti-symmetric. we can have (4, 2) but we cannot have (2, 4)

It is non-transitive. If we have (x, y) = (4, 2) and (y, z) = (2, 1), (x, z) = (4, 1) but

it doesn’t satisfy the statement x = 2y as (4) ≠ 2(1)

1. (a) It is reflexive. Because 0 = 0 🡪 a - a = b - b 🡪 ((a, b),(a, b)) ∈ R

It is symmetric. ((a, b),(c, d)) 🡪 a - c = b - d 🡪 c - a = d - b -> ((c, d),(a, b))

It is transitive. Have ((a, b),(c, d)) 🡪 a - c = b - d , ((c, d),(e, f)) 🡪 c - e = d – f.

Then, (a - c) + (c - e) = (b - d)+(d - f) 🡪 a - e = b - f 🡪 ((a, b),(e, f))

(b) f(x, y) = x - y. Then we have: f(a, b) = f(c, d) 🡪 a - b = c - d

🡪 a - c = b – d

🡪 ((a, b),(c, d))

(c) ((a, b), (1, 1)) 🡪 a - 1 = b - 1 🡪 a = b 🡪 The class is {(a, a)}

🡪 2 elements: (2, 2) and (3, 3)

(d) There are infinite classes with infinite number of elements. Each class is the set

of tuples {(a, b)} where the difference between a and b is constant.

1. (a) It is reflexive because (a, a) ∈ R4.

It is symmetric because (a, b) ∈ R4 = (b, a) ∈ R4

It is transitive because if a and b is in the same building, b and c is in the same

building, then a and c is in the same building.

Therefore, it is an equivalent relation.

R4 = {(a, b) | lives in the same building}

If R is the refinement of S. Subset or refinement of same building or same unit or same floor.

(b) It is reflexive because (a, a) ∈ R5.

It is symmetric because (a, b) ∈ R5 = (b, a) ∈ R5

It is transitive because (a = b) = (b = c). a and b graduated from the same high

school, b and c graduated from the same high school, so a and c graduated from the

same high school.

Therefore, it is an equivalent relation.

R5 = {(a, b) | graduated from the same high school}

If R is the refinement of S. Subset or refinement of same high school, same college, or same year.

1. (a) it is reflexive 🡪 x/x = 1 ∈ Z

It is anti-symmetric 🡪 x/y does not always mean y/x unless x = y

It is transitive 🡪 if we have (x, y) = (1, 2) and (y, z) = (2, 3) then we have (x, z) = (1, 3). x/z = 1/3 which is ∈ Z.

(x, y) ∈ R6 if and only if x/z ∈ Z is partial ordered set.

(b)It is reflexive 🡪 x – x = 0 ∈ Z

It is symmetric 🡪 x – y ∈ Z 🡪 -(x-y) ∈ Z 🡪 y – x ∈ Z

It is transitive 🡪 have x – y ∈ Z & y – z ∈ Z , x – z

= x – y + y – z

=(x – y) +(y – z) ∈ Z

(x, y) ∈ R7 if and only if x – y ∈ Z is equivalent relation

[2]R7 = Z+ and [π] R7 = { π – 3 + n | n ∈ N}